Calculus 1
Midterm Exam
September 30, 2024 (18:30 – 20:30)

university of groningen

Please read the instructions!

1) Prove using the (ε, δ)-definition that $\lim_{x \to 0} \sqrt{1 - x^2} = 1$.

2) Apply l'Hospital's rule to evaluate the following limit: $\lim_{x \to \infty} \frac{x^{\ln x}}{(\ln x)^x}$.
Indicate which rules of differentiation are being used in each step.

3) Use implicit differentiation to find an equation of the tangent line to the logarithmic spiral $\arctan \frac{y}{x} = \ln \sqrt{x^2 + y^2}$ at the point $\left(\frac{\sqrt{3}}{2} e^{\pi/6}, \frac{1}{2} e^{\pi/6} \right)$.

4) Prove the following statement for every positive integer n.

If $r > 1$, then there exists a $c \in (1, r)$ such that $\frac{1}{n} \sum_{k=0}^{n-1} r^k = c^{n-1}$.

5) Compute the degree 2 Taylor polynomial of $f(x) = \frac{x - 3}{\sqrt{x^2 + 3}}$ around $x = -1$.

6) Find the value $f(2)$ given that the graph of f passes through the point $(1, 4)$ and the slope of the tangent line at $\big(x, f(x)\big)$ is $2x + \ln x$.

Instructions

- **write your name and student number on the top of each sheet of writing paper!**
- use the writing (lined) and scratch (blank) paper provided, raise your hand if you need more paper
- start each question on a new page
- use a pen with black or blue ink
- do not use any kind of correcting fluid or tape
- any rough work should be crossed through neatly so it can be seen
- this is a closed-book exam (books & notes are not permitted)
- you may use your own handwritten formula sheet (a 2-sided A4 paper) – the invigilators will check these
- you are also allowed to use a simple pocket calculator
- programmable/graphing calculators are not allowed
- your work should be clearly and logically structured
- **explain your reasoning using words**
- show all your calculations, an answer without any computation will not be rewarded
- each problem is worth 15 points
- upon completion[1] submit your worksheets at the front desk

[1] At the end of the exam or after you finished whichever is sooner.